Recovery of Quantized Compressed Sensing Measurements

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ABSTRACT

The mathematical theory of Compressed Sensing has been applied to various engineering areas ranging from one-pixel cameras,[1] to range imaging[2] and medical ultrasound imaging,[3] to name a few. The theory developed within CS suggests that one can achieve perfect reconstruction of a signal $\mathbf{x} \in \mathbb{R}^N$ from a small number of random measurements $\mathbf{y} \in \mathbb{R}^M$, far below the typical Shannon-Nyquist sampling limit. Recovery from the compressed measurements $\mathbf{y}$ is possible by exploiting the sparsity of the signal, when expressed in an appropriate dictionary $\mathbf{D}$ according to $\mathbf{x} = \mathbf{D}\mathbf{s}$, where the sparsity is measured by the non-zero element counting pseudo norm $\|\mathbf{s}\|_0 = K$. The acquisition process can be formulated according to $\mathbf{y} = \mathbf{\Psi}\mathbf{x} = \mathbf{\Psi}\mathbf{D}\mathbf{s}$, where $\mathbf{\Psi}$ is the sampling operator matrix.

Despite the recent explosion of CS sampling architectures, the majority of literature studies the scenario where a sufficiently large number of randomly encoded measurements are available at the decoder for signal reconstruction. Although signal sampling and reconstruction are crucial, systems that implement CS must use a finite number of distinct symbols via a quantization process in order to efficiently store and transmit the random linear measurements. From the CS perspective, the decoder is now presented with $\hat{\mathbf{y}} = \mathcal{Q}(\mathbf{y})$, where the nonlinear operator $\mathcal{Q} : \mathbb{R} \to 2^P$ models the process of mapping the set of real numbers to a set of specific elements indexed by P bits. The process of recovery can then be expressed according to an l_0 minimization given by:

$$\min \|\mathbf{s}\|_0 + \lambda \|\hat{\mathbf{y}} - \mathbf{\Psi}\mathbf{D}\mathbf{s}\|_2 \ . \tag{1}$$

The necessity of quantization for storage and transmission has motivated the investigation of the effects of quantization on the recovery capabilities of (1). The challenges associated with analyzing and overcoming the limitations of quantization are directly related to the non-linear nature of the process. The problem has been studied for different quantization schemes, ranging form uniform[4] to sigma-delta[5] to the extreme case of 1-bit C S,[6,7] where the recovery algorithm is presented merely with the signs of the random measurements.

In this work, we propose a novel formulation of CS recovery where the effects of quantization are taken into account during reconstruction. Formally, we consider a non-linear mapping function of the sparse signal $\mathbf{s}$ given by $\mathbf{\Psi}(\mathbf{s}) = \mathcal{Q}(\mathbf{G}\mathbf{D}\mathbf{s})$. In this case, the recovery program can be expressed according to:

$$\min \|\mathbf{s}\|_0 + \lambda \|\hat{\mathbf{y}} - \mathbf{\Psi}(\mathbf{s})\|_2 \ . \tag{2}$$

Recovery in the presence of the non-linear function $\mathbf{\Psi}$ can be achieve by solving a modified greedy minimization problem, the Orthogonal Matching Pursuit (OMP), with additional constraints on the consistency of the recovered signal with respect to its quantized counterpart. The superior recovery capabilities of the proposed approach are illustrated in Figure 1 where a comparison between the traditional and the proposed schemes is presented for the (a) 4 bits and (b) 8 bits per measurement scenarios. The figure showcases the approximation error between a compressible signal $\mathbf{x} \in \mathbb{R}^{100\times 1}$ and the solution produced by solving (1) and (2) as a function of the sampling rate. We consider signals that can be sparsely represented using 2 elements from a DCT dictionary. The significant benefits associated with the recovery using the modified OMP are clear in both cases and under all sampling rates. We should also point out that the additional performance gains are accompanied with a marginal increase in decoding complexity.

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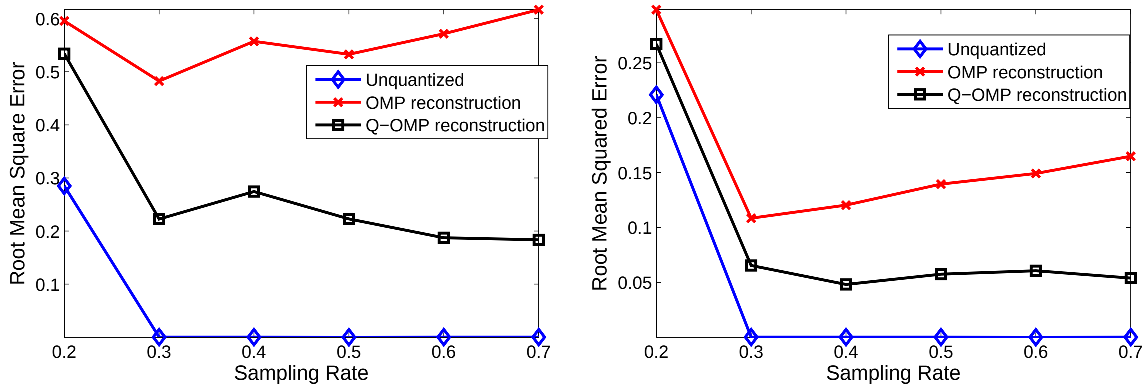

(a) 4bits per measurement

(b) 8bits per measurement

Figure 1: Recovery of compressible signals from unquantized and quantized measurements using the traditional OMP and the Quantization aware OMP for the (a) 4 bits per measurement and the (b) 8 bits per measurement scenarios

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